LECTURE 101

- KINEMATICS
- RECTILINEAR MOTION
- CURVILINEAR MOTION
- RELATIONSHIP BETWEEN LINEAR AND ROTATIONAL VARIABLES
- PROJECTILE MOTION

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Nomenclature		**English unit**	**SI unit**
a	acceleration	ft/sec^2	m/s^2
g	gravitational	ft/sec^2	m/s^2
r	position	ft	m
r	radius	ft	m
s	distance	ft	m
t	time	sec	s
v	velocity	ft/sec^2	m/s

Symbols

α	angular acceleration	rad/sec^2	rad/s^2
θ	angular position	rad	rad
ω	angular velocity	rad/sec	rad/s

Subscripts

0	initial
f	final
n	normal
r	radial
θ	tangential
e_θ	transverse

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INTRODUCTION TO KINEMATICS

Dynamics is the study of moving objects. The subjects is divided into kinematics and kinetics. Kinematics is the study of a body's motion independent of the force on body. It is a study of the geometry of motion with consideration of the causes of motion. Kinematics deals only with relationships among position, velocity, acceleration, and time.

A body in motion can be considered a particle if rotation of the body is absent or insignificant. A particle does not passes rotational kinetic energy. All parts of a particle have the same instantaneous displacement, velocity, and acceleration.

A *rigid body* does not deform when loaded and can be considered a combination of two or more particles that remain at a fixed, finite distance from each other. At any given instant, the parts (particles) of a rigid body can have different displacements, velocities, and accelerations if the body has rotational as well as translational motion.

If r is the position vector of particle, the instantaneous velocity and acceleration are

$$r \qquad \text{[position]} \tag{14.1}$$

$$v = \frac{dr}{dt} \qquad \text{[velocity]} \tag{14.2}$$

$$a = \frac{dv}{dt} = \frac{d^2r}{dt^2} \qquad \text{[acceleration]} \tag{14.3}$$

RECTILINEAR MOTION

A *rectilinear system* is one in which particles move only in straight lines. (Another name is *linear system*.) The relationships among position, velocity, and acceleration for a linear system are given by Eqs. 14.4 through 14.6.

$$s(t) = \int v(t)dt = \int a(t)dt^2 \tag{14.4}$$

$$v(t) = \frac{ds(t)}{dt} = \int a(t)dt \tag{14.5}$$

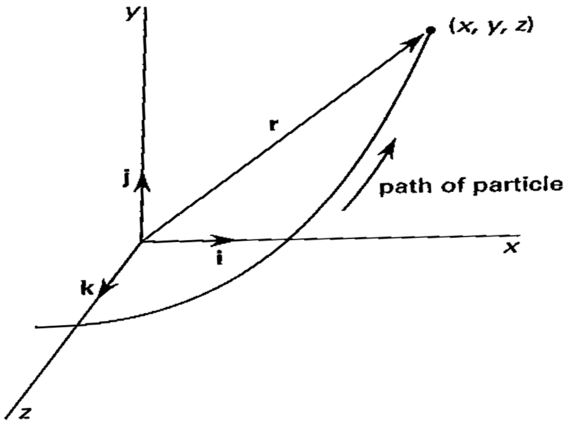
$$a(t) = \frac{dv(t)}{dt} = \frac{d^2s(t)}{dt^2} \tag{14.6}$$

Rectangular coordinates

The position of a particle is specified with reference to a coordinate system. Three coordinates are necessary to identify the position in three-dimensional space; in two dimensions, two coordinates are necessary. A coordinates can represent a linear position, as in the rectangular coordinate system, or it can represent an angular position, as in the polar system.

Consider the particle shown in Fig. 14.1 its position, as well as its velo9city and acceleration, can be specified in tree primary forms: vector form, rectangular coordinate form, and unit vector form.

Figure 14.1 Rectangular Coordinates



The *vector form* of the particle's position is r, where the vector r has both magnitude and direction. The *retangular coordinate form* is (x, y, z). The *unit vector form* is

$$r = xi = yk = zk \qquad 14.7$$

The velocity and acceleration are the first two derivatives of the position vector, as shown in eqs. 14.8 and 14.9.

$$v = \frac{dr}{dt}$$

$$v = \dot{x}i + \dot{y}j + \dot{z}k \qquad 14.8$$

$$a = \frac{dv}{dt} = \frac{d^2r}{dt^2}$$

$$a = \ddot{x}i + \ddot{y}j + \ddot{z}k \qquad 14.9$$

Constant Acceleration

Acceleration is constant in many cases, such as free falling body with constant acceleration g. If the acceleration is constant, the acceleration term can be taken out of the integrals in Eqs. 14.4. The initial distance from the origin is s_0; the initial velocity is a constant, v_0; and a constant acceleration is denoted a_0.

$$a(t) = a_0 \qquad 14.10$$

$$v(t) = a_0 \int dt = v_0 + a_0 t \qquad 14.11$$

$$s(t) = a_0 \iint dt^2$$

$$(t) = s_0 + v_0 t + \frac{a_0 t^2}{2} \qquad 14.12$$

$$v^2 t = {v_0}^2 + 2a_0(s - s_0) \qquad 14.13$$

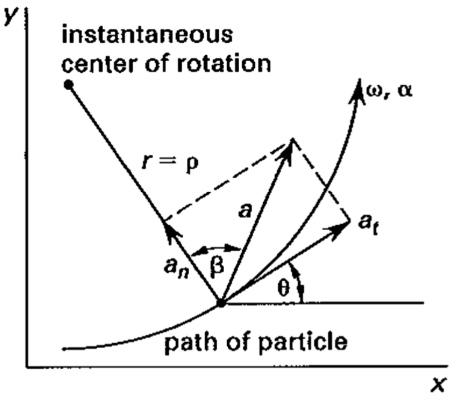
CURVILINEAR MOTION

Curvilinear motion describes the motion of a particle along a path that is not a straight line. Special examples of curvilinear motion include plane circular motion and projectile motion. For particles traveling along curvilinear paths, the position, velocity, and acceleration may be specified in rectangular coordinates as they were for rectilinear motion, or it may be more convenient to express the kinematic variables in terms of other coordinate systems (e.g., polar coordinates).

Transverse and Radial Components

In polar coordinates, the position of a particle is described by a radius, r, and an angle, θ. The position may also be expressed as a vector of magnitude r and direction specified by unit vector e_r. Since the velocity of a particle is not usually directed radially out from the center of the coordinate system, it can be divided into two components, called *radial and transverse*, which are parallel and perpendicular, respectively, to the unit radial vector. Figure 14.2 illustrates the radial and transverse components of velocity in a polar coordinate system, and the unit radial and transverse vectors, e_r and e_θ, used in the vector forms of the motion equations.

Figure 14.3 Tangential and Normal Coordinates



$$r = re_r \qquad \text{[position]} \tag{14.14}$$

$$v = v_r e_r + v_\theta e_\theta$$

$$v = \dot{r}e_r + r\dot{\theta}e_\theta \qquad \text{[velocity]} \tag{14.15}$$

$$a = a_r\, e_r + a_\theta e_\theta$$

$$a = \left(\ddot{r} - r\dot{\theta}^2\right)e_r + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)e_\theta \qquad \text{[acceleration]} \tag{14.16}$$

Tangential and Normal Components

A particle moving in a curvilinear path will have instantaneous linear velocity and linear acceleration. These linear variables will be directed tangentially to the path, and, therefore, are known as *tangential velocity*, v_t, and *tanential acceleration*, a_t, respectively. The force that constrains the particle to the curved path will generally be directed toward the center of rotation, and the particle will experience an inward acceleration perpendicular to the tangential velocity and acceleration, known as the *normal acceleration*, a_n. The resultant acceleration, a, is the vector sum of the tangential components of acceleration are illustrated in Fig. 14.3. the vectors e_n and e_t are normal and tangential to the path, respectively. ρ is the principal *radius of curvature*.

$$v = v_t e_t \tag{14.17}$$

$$a = \left(\frac{dv_t}{dt}\right)e_t + \left(\frac{v^2{}_t}{\rho}\right)e_n \tag{14.18}$$

Particle Circular Motion

Plane circular motion (also known as *rotational particle motion*, *angular motion*, or *circular motion*) is motion of particle around a fixed circular path. The behavior of a rotating particle is defined by its angular position, θ, angular velocity, ω, and angular acceleration, α. These variables are analogous to the s, v, and α variables for linear systems. Angular variables can be substituted one-for-one in place of linear variables in most equations.

θ [angular position] *14.19*

$\omega = \frac{d\theta}{dt}$ [angular velocity] *14.20*

$\alpha = \frac{d\omega}{dt}$

$\alpha = \frac{d^2\theta}{dt^2}$ [angular acceleration] *14.21*

RELATIONSHIPPS BETWEEN LINEAR AND ROTATIONAL VARIABLES

$s = r\theta$ *14.22*

$v_t = r\omega$ *14.23*

$a_t = r\alpha = \frac{dv_t}{dt}$ *14.24*

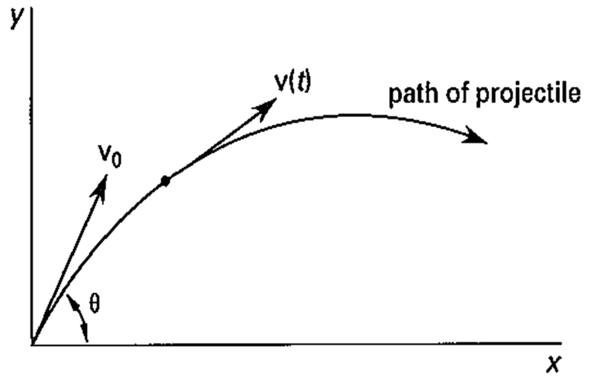
$\alpha_n = \frac{v^2{}_t}{r} = r\omega^2$ *14.25*

PROJECTILE MOTION

A projectile is placed into motion by an initial impulse. (Kinematics deals only with dynamics during the flight. The force acting on the projectile during the lunch phase is covered in kinematics.) Neglecting air drag, once the projectile is in motion, it is acted upon only by the downward gravitational acceleration (i.e., its own weight). Thus, projectile motion is a special case of motion under constant acceleration.

Consider a general projectile set into motion at an angle of θ from the horizontal plane, and initial velocity v_0, as shown in Fig. 14.4. In the absence of air drag, the following rules apply to the case of travel over a horizontal plane.

Figure 14.4 Projectile Motion



- The trajectory is parabolic.
- The impact velocity is equal to initial velocity, v_0.
- The range is maximum when $\theta = 45°$.
- The time for the projectile to travel from the launch point to the apex is equal to the time to travel from apex to impact point.
- The time for the projectile to travel from the apex of its flight path to impact is the same time an initially stationary object would take to fall straight down from that height.

The following solutions to most common projectile problems are derived from the laws of uniform acceleration and conservation of energy.

$$a_x = 0 \qquad 14.26$$

$$a_y = -g \qquad 14.27$$

$$v_x = v_{x0} = v_0 \cos\theta \qquad 14.28$$

$$v_y = v_{y0} - gt = v_0 \sin\theta - gt \qquad 14.29$$

$$x = v_{x0}t = v_o t \cos\theta \qquad 14.30$$

$$y = v_{y0}t - \frac{1}{2}gt^2 = v_o t \sin\theta - \frac{1}{2}g\,t^2 \qquad 14.31$$

SAMPLE PROBLEMS

Problem 1-3 refer to a particle whose curvilinear motion is represented by the equations $s = 20t + 4t^2 - 3t^3$.

Problem 1

What is the particle's initial velocity?

(A) 20 m/s

(B) 25 m/s

(C) 30 m/s

(D) 32 m/s

Solution

$$v = \frac{ds}{dt} = 20 + 8t - 9t^2$$

At $t = 0$

$$v = 20 + 8(0) - 9(0)^2$$

$$\boxed{v = 20\ m/s}$$

Problem 2

What is the acceleration of the particle at time $t = 0$?

(A) 2 m/s^2

(B) 3 m/s^2

(C) 5 m/s^2

(D) 8 m/s^2

Solution

$$a = \frac{d^2s}{dt^2} = 8 - 18t$$

At $t = 0$,

$$\boxed{a = 8\ m/s^2}$$

Problem 3

What is maximum speed reached by the particle?

(A) 21.8 m/s

(B) 27.9 m/s

(C) 34.6 m/s

(D) 48.0 m/s

Solution

$$v = 20 + 8t - 9t^2$$

$$\frac{dv}{dt} = 8 - 18t = 0$$

$$t = \frac{8}{18}s = 0.444\ s$$

$$v_{max} = 20 + (8)(0.444\ s) - (9)(0.444\ s)^2$$

$$\boxed{v_{max} = 21.8\ m/s}$$

Problem 4

Choose the equation that best represents a rigid body or particle under constant acceleration.

(A) $a = 9.81\ {}^{m}/_{s^2} + v_0/t$

(B) $v = v_0 + a_0 t$

(C) $v = v_0 + \int_0^t a(t)dt$

(D) $a = v^2{}_t/r$

Solution

Choice (B) is the expression for the velocity of a linear system under constant acceleration.

$$v(t) = a_0 \int dt = v_0 + a_0 t$$

The other answer choices can be eliminated. Choice (A) is an expression for acceleration that varies with time; choice (C) is an expression for velocity with a generalized time-varying acceleration. The expression in choice (D) relates tangential and normal accelerations, respectively, along a curved path, to the tangential velocity. For a generalized curved path, these acceleration are not constant.

Problem 5

A roller coaster train climbs a hill with a constant gradient. Over a 10 s period, the acceleration is constant at 0.4 m/s^2 , and the average velocity of the train is 40 km/hr. Find the final velocity.

(A) 9.1 m/s

(B) 11.1 m/s

(C) 13.1 m/s

(D) 15.1 m/s

Solution

Use the constant acceleration equations for straight line motion.

$$v_{ave} = \left(40\,\frac{km}{hr}\right)\left(1000\,\frac{m}{km}\right)\left(\frac{1}{60\ min/hr}\right)\left(\frac{1}{60\ s/min}\right)$$

$$v_{ave} = 11.11\ m/s$$

The distance traveled in 10 s is

$$\left(11.11\,\frac{m}{s}\right)(10\ s) = 111.1\ m$$

$$s(t) = s_0 + v_0 t + \frac{1}{2}a_0 t^2$$

$$v_0 = \frac{s(t) - s_0 - \frac{1}{2}a_0 t^2}{t}$$

$$v_0 = \frac{111.1\ m - 0 - \left(\frac{1}{2}\right)\left(0.4\frac{m}{s^2}\right)(10\ s)^2}{10\ s}$$

$$v_0 = 9.11\ m/s$$

$$v_f = v_0 + a_0 t$$

$$v_f = 9.11\,\frac{m}{s} + \left(0.4\,\frac{m}{s^2}\right)(\ 10\ s)$$

$$\boxed{v_f = 13.11\ m/s}$$

alternate Solution

$$v = v_0 + a_0 t$$

$$v_{ave} = \frac{\Delta s}{\Delta t} = \frac{1}{t_f - t_0}\int_{t_0}^{t_f}(v_0 + a_0 t)dt$$

$$v_{ave} = \int_{t_0}^{t_f}\left(\frac{1}{t_f - t_0}\right)\left(v_0 t + \frac{1}{2}a_0 t^2\right)$$

$$v_{ave} = \left(\frac{1}{t_f - t_0}\right)\left(v_0 t_f + \frac{1}{2} a_0 {t^2}_f - v_0 t_0 - \frac{1}{2} a_0 {t^2}_0\right)$$

$Arbitrarily, let\ t_0 = 0\ and\ t_f = 10.$

$$v_{ave} = \left(\frac{1}{t_f}\right)\left(v_0 t_f + \left(\frac{a_0}{2}\right) {t^2}_f\right)$$

$$v_{ave} = v_0 + \frac{a_0 t_f}{2}$$

$$v_0 = v_{ave} - \frac{a_0 t_f}{2}$$

$$v_0 = 11.11\frac{m}{s} - \left(\frac{0.4\frac{m}{s^2}}{2}\right)(10\ s)$$

$$v_{ave} = 9.11\ m/s$$

$$v_{ave} = \frac{v_0 + v_f}{2}$$

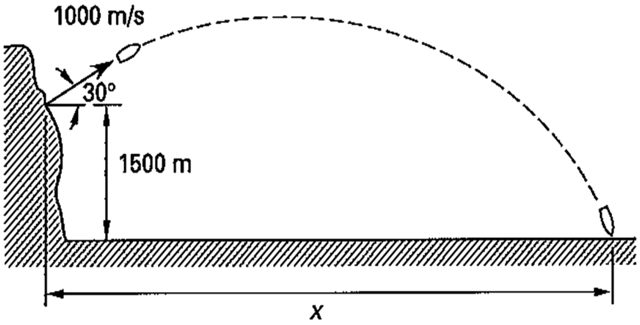
$$v_f = 2v_{ave} - v_0$$

$$v_f = (2)\left(11.11\frac{m}{s}\right) - 9.11\frac{m}{s}$$

$$\boxed{v_f = 13.11 m/s}$$

Problem 6

A projectile is fired from a cannon with an initial velocity of 1000 m/s and at an angle of 30° form the horizontal. What distance from the cannon will the projectile strike the ground if the point of impact is 1500 m below the point of release?



(A) 8200 m	(C) 78 200 m
(B) 67 300 m	(D) 90 800 m

Solution

$$y = v_0 t \sin\theta - \frac{gt^2}{2}$$

$$\frac{g}{2}t^2 - v_0 t \sin\theta + y = 0$$

$y = -1500$ m since it is below the launch plane.

$$\left(\frac{9.81\frac{m}{s^2}}{2}\right)t^2 - \left(1000\frac{m}{s}\right)t\sin 30° - 1500\,m = 0$$

$$\left(4.905\frac{m}{s^2}\right)t^2 - \left(500\frac{m}{s}\right)t - 1500\,m = 0$$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$t = \frac{500 \pm \sqrt{(-500)^2 - 4(4.905)(1500)}}{2(4.905)}$$

$$t = +104.85\,s, \qquad 2.9166\ \ s$$

$$x = v_0 t \cos\theta$$

$$x = \left(1000\frac{m}{s}\right)\left(104\frac{m}{s}\right)\cos 30°$$

$$\boxed{x = 90\,803\,m}$$